

## Step-by-Step Integral Evaluation

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1  x, y, z = symbols('x,y,z')
2  f = Symbol('f(x,y,z)')
3
4  # Define limits of integration
5  x_llim = 0
6  x_ulim = 2
7  y_llim = 0
8  y_ulim = 3
9  z_llim = 0
10 z_ulim = 4
11
12 print(r'\begin{align*}')
13
14 # Notice how I define f as a symbol, then later as an actual function
15 left = Integral(f, (x, x_llim, x_ulim), (y, y_llim, y_ulim), (z, z_llim, z_ulim))
16 f = x*y + y*sin(z) + cos(x+y)
17 right = Integral(f, (x, x_llim, x_ulim), (y, y_llim, y_ulim), (z, z_llim, z_ulim))
18 print(latex(left) + '&=' + latex(right) + r'\\')
19
20 # For each step, I move limits from an outer integral to an inner, evaluated
21 # integral until the outer integral is no longer needed
22 right = Integral(Integral(f, (z, z_llim, z_ulim)).doit(), (x, x_llim, x_ulim),
23                  (y, y_llim, y_ulim))
24 print('&=' + latex(right) + r'\\')
25
26 right = Integral(Integral(f, (z, z_llim, z_ulim), (y, y_llim, y_ulim)).doit(),
27                  (x, x_llim, x_ulim))
28 print('&=' + latex(right) + r'\\')
29
30 right = Integral(f, (z, z_llim, z_ulim), (y, y_llim, y_ulim),
31                  (x, x_llim, x_ulim)).doit()
32 print('&=' + latex(right) + r'\\')
33
34 print('&=' + latex(N(right)) + r'\\')
35
36 print(r'\end{align*}')

```

$$\begin{aligned}
\int_0^4 \int_0^3 \int_0^2 f(x, y, z) \, dx \, dy \, dz &= \int_0^4 \int_0^3 \int_0^2 xy + y \sin(z) + \cos(x + y) \, dx \, dy \, dz \\
&= \int_0^3 \int_0^2 4xy - y \cos(4) + y + 4 \cos(x + y) \, dx \, dy \\
&= \int_0^2 18x - 4 \sin(x) + 4 \sin(x + 3) - \frac{9}{2} \cos(4) + \frac{9}{2} \, dx \\
&= 4 \cos(3) + 4 \cos(2) - 4 \cos(5) - 9 \cos(4) + 41 \\
&= 40.1235865133293
\end{aligned}$$